E5.1 / UNITS OF MEASURE

Mensuration notes

Mensuration is the measurement of lengths, areas and volumes. The metric system uses related units that convert by powers of 10.

Quantity	Meaning	Common units
Length	Distance between two points	mm, cm, m, km
Mass	The amount of matter in an object	g, kg
Area	The amount of surface covered	mm^2, cm^2, m^2, km^2
Volume	The amount of three-dimensional space occupied	mm^3, cm^3, m^3
Capacity	The amount a container can hold when full	mL, L

Metric conversions

Length 10 mm = 1 cm · 100 cm = 1 m · 1000 m = 1 km

Mass 1000 g = 1 kg · **Capacity** 1000 mL = 1 L

Volume and capacity 1 cm^3 = 1 mL · 1000 cm^3 = 1 L · 1 m^3 = 1000 L

If 1 large length unit equals **k** small units, multiply by **k** for length, **k^2** for area and **k^3** for volume when converting large units to small units. Divide by the same factor in the reverse direction.

Worked examples / Length and mass

Convert 3.45 km to metres and 625 g to kilograms.

3.45 × 1000 = **3450 m** · 625 ÷ 1000 = **0.625 kg**

Worked examples / Area

Convert 6500 cm^2 to m^2 and 0.04 km^2 to m^2.

6500 ÷ 100^2 = **0.65 m^2** · 0.04 × 1000^2 = **40 000 m^2**

Worked examples / Volume and capacity

Convert 250 000 mm^3 to cm^3 and 2.4 m^3 to litres.

250 000 ÷ 10^3 = **250 cm^3** · 2.4 × 1000 = **2400 L**

Unit names · millimetre (mm), centimetre (cm), metre (m), kilometre (km), gram (g), kilogram (kg), millilitre (mL), litre (L).

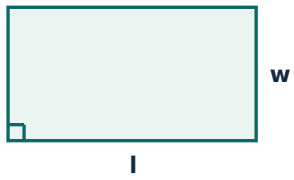
E5.2 / AREA AND PERIMETER

Area and perimeter

Perimeter is the total distance around a shape. Add all its outside side lengths.

Area is the amount of space inside a flat shape. Give area in square units.

Base and perpendicular height The base is the chosen reference side. The height is measured at 90° to the base or its extension. Match your units before calculating.

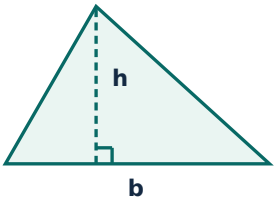


Rectangle

A four-sided shape with four right angles. The length is l and the width is w.

Area = l × w · Perimeter = 2(l + w)

Example For l = 8 cm and w = 5 cm, area = 8 × 5 = **40 cm²** and perimeter = 2(8 + 5) = **26 cm**.

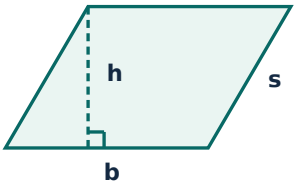


Triangle

A three-sided shape. The base is b and the perpendicular height is h.

Area = ½bh · Perimeter = sum of the three sides

Example A 5–5–6 cm triangle has height 4 cm to the 6 cm base. Area = ½ × 6 × 4 = **12 cm²** and perimeter = 5 + 5 + 6 = **16 cm**.

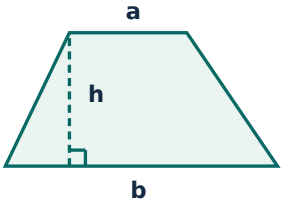


Parallelogram

A four-sided shape with two pairs of parallel sides. The base is b, the adjacent side is s and the perpendicular height is h.

Area = bh · Perimeter = 2(b + s)

Example For b = 9 cm, s = 5 cm and h = 4 cm, area = 9 × 4 = **36 cm²** and perimeter = 2(9 + 5) = **28 cm**.



Trapezium

A four-sided shape with one pair of parallel sides, a and b. Their perpendicular separation is h.

Area = ½(a + b)h · Perimeter = sum of all four sides

Example Parallel sides are 6 cm and 12 cm. The height is 4 cm and the other sides are 5 cm each. Area = ½(6 + 12) × 4 = **36 cm²** and perimeter = 6 + 12 + 5 + 5 = **28 cm**.

Formula list for E5.2 · The area formula for a triangle is given. Learn the remaining formulas on this page.

E5.3 / CIRCLES, ARCS AND SECTORS

Circles, arcs and sectors

Circle A round shape whose boundary points are all the same distance from its centre.

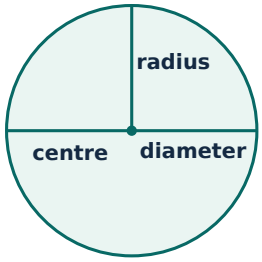
Radius (r) A segment from the centre to the circumference, or its length.

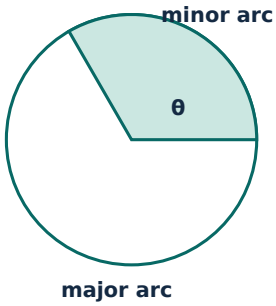
Diameter (d) A segment joining two boundary points through the centre. Its length is 2r.

Circumference The perimeter of a circle. The number π is the ratio of circumference to diameter.

Arc Part of the circumference. A minor arc is shorter than a semicircle. A major arc is longer.

Sector The region bounded by two radii and an arc. A minor sector has central angle less than 180°. A major sector has central angle greater than 180°. A 180° sector is a semicircle.





Circle and sector formulas

Circumference = $2\pi r = \pi d$
Circle area = πr^2

Arc length = $(\theta \div 360°) \times 2\pi r$
Sector area = $(\theta \div 360°) \times \pi r^2$

θ is the angle between the radii measured through the sector.

Worked example / A whole circle

A circle has diameter 14 cm. Find its circumference and area.

Radius = 14 ÷ 2 = 7 cm. Circumference = 14π = **14π cm**.

Area = $\pi \times 7^2$ = **49π cm²**.

Worked example / A minor and a major sector

In a circle of radius 9 cm, two radii form a minor sector with angle 120°. Find both arc lengths and both sector areas.

Minor 120° ÷ 360° = ⅓. Arc length = ⅓ × 18π = **6π cm**.
Area = ⅓ × 81π = **27π cm²**.

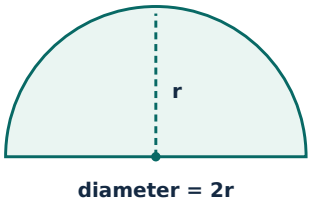
Major Angle = 360° − 120° = 240°. Arc length = ⅔ × 18π = **12π cm**.
Area = ⅔ × 81π = **54π cm²**.

An answer **in terms of π** is exact. Leave π in the final answer when asked. A sector's perimeter includes its arc and both radii. For the minor sector above, it is **$(6\pi + 18)$ cm**.

E5.3 / CIRCLES, ARCS AND SECTORS

Semicircles and quadrants

A full circle has a central angle of 360°. Use the fraction of a full turn to find the area and arc length. For the perimeter, include the straight edges too.



Semicircle / half a circle

Its central angle is 180°. Its boundary is made of a half-circle arc and one diameter.

Area = ½ × πr² = ½πr²
Arc length = ½ × 2πr = πr
Perimeter = πr + 2r

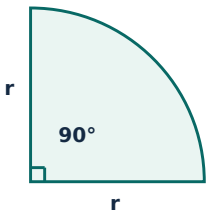
Worked example / Semicircle given its diameter

A semicircle has diameter 14 cm. Find its area and perimeter in terms of π.

Radius = 14 ÷ 2 = 7 cm.

Area = ½ × π × 7² = **24.5π cm²**.

Arc length = 7π cm. Add the diameter. Perimeter = **(7π + 14) cm**.



Quadrant / one quarter of a circle

Its central angle is 90°. Its boundary is made of a quarter-circle arc and two perpendicular radii.

Area = ¼ × πr² = ¼πr²
Arc length = ¼ × 2πr = ½πr
Perimeter = ½πr + 2r

Worked example / Quadrant given its radius

A quadrant has radius 6 cm. Find its area and perimeter in terms of π.

Area = ¼ × π × 6² = **9π cm²**.

Arc length = ½ × π × 6 = 3π cm. Add the two radii.
Perimeter = 3π + 6 + 6 = **(3π + 12) cm**.

E5.4 / SURFACE AREA AND VOLUME

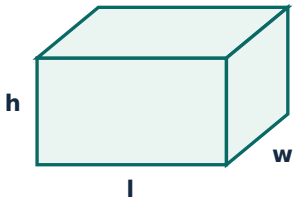
Cuboids, prisms and cylinders

Surface area The total area of the outside surfaces. Use square units. Curved surface area counts only a curved surface.

Volume The space a solid occupies. Use cubic units.

Face, edge and vertex A face is a flat surface. An edge is where two surfaces meet. A vertex is a corner.

Cross-section The shape made by a flat cut through a solid. A uniform cross-section has the same shape and size along the length.

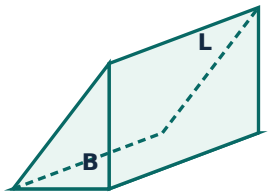


Cuboid

A box-shaped solid with six rectangular faces. Its dimensions are length l, width w and height h.

Volume = lwh
Surface area = 2(lw + lh + wh)

Example For a 6 cm by 4 cm by 3 cm cuboid, volume = 6 × 4 × 3 = **72 cm³** and surface area = 2(24 + 18 + 12) = **108 cm²**.

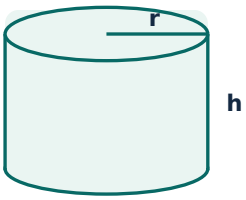


Prism

A solid with a uniform cross-section. For a right prism, the length L is perpendicular to the ends. Let B be one end's area and P its perimeter.

Volume = BL
Surface area of a right prism = 2B + PL

Example A right prism has triangular ends with sides 3, 4 and 5 cm. For L = 10 cm, B = ½ × 3 × 4 = 6 cm² and P = 12 cm. Volume = 6 × 10 = **60 cm³**. Surface area = 2 × 6 + 12 × 10 = **132 cm²**.



Cylinder

A right circular cylinder has two identical circular ends of radius r. Its height h is the perpendicular distance between the ends.

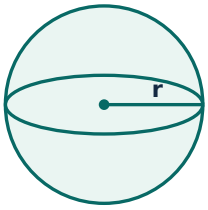
Volume = $\pi r^2 h$ · Curved surface area = $2\pi rh$
Total surface area = $2\pi rh + 2\pi r^2$ for a closed cylinder

Example For r = 3 cm and h = 8 cm, volume = **72π cm³** and curved area = **48π cm²**. Add $2\pi \times 3^2 = 18\pi$ cm² for the ends. Total surface area = **66π cm²**.

E5.4 / SURFACE AREA AND VOLUME

Spheres, cones and pyramids

Base means the reference face of a solid. The **apex** is the point where the sides of a cone or pyramid meet. **Perpendicular height** is measured at 90° to the base. **Slant height** is measured along the surface from the apex to the base rim of a right cone, or along a triangular face of a regular pyramid to the midpoint of its base edge.

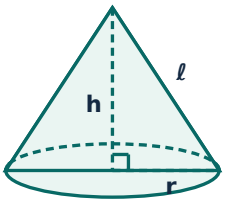


Sphere

A solid whose surface points are all a distance r from its centre. Its diameter is 2r.

Surface area = $4\pi r^2$ · Volume = $\frac{4}{3}\pi r^3$

Example For r = 3 cm, surface area = $4\pi \times 3^2$ = **36π cm²** and volume = $(4 \div 3)\pi \times 3^3$ = **36π cm³**.

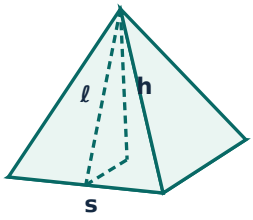


Cone

A right circular cone has a circular base of radius r. Its apex is directly above the centre. Height is h and slant height is ℓ.

Volume = $\frac{1}{3}\pi r^2 h$ · Curved surface area = $\pi r\ell$
Total surface area = $\pi r\ell + \pi r^2$ · $\ell^2 = r^2 + h^2$

Example For r = 3 cm and h = 4 cm, $\ell = \sqrt{(3^2 + 4^2)} = 5$ cm. Volume = $\frac{1}{3}\pi \times 3^2 \times 4$ = **12π cm³**. Curved area = $\pi \times 3 \times 5$ = **15π cm²**. Total surface area = 15π + 9π = **24π cm²**.



Pyramid

A solid with a polygon base and triangular faces meeting at an apex. A polygon is a closed shape with straight sides. Let B be the base area and h the perpendicular height.

Volume = $\frac{1}{3}Bh$
Surface area = base area + areas of all triangular faces
For a right square pyramid with base side s, surface area = $s^2 + 2s\ell$ and $\ell^2 = h^2 + (s \div 2)^2$.

Example A right square pyramid has base side 10 cm and height 12 cm. Its apex is above the base centre, so $\ell = \sqrt{(12^2 + 5^2)} = 13$ cm. Volume = $\frac{1}{3} \times 100 \times 12$ = **400 cm³**. Surface area = $100 + 4(\frac{1}{2} \times 10 \times 13)$ = **360 cm²**.

Use the perpendicular height for cone and pyramid volume. Use the slant height when finding the curved area of a cone or the area of a triangular face of a right square pyramid.

E5.5 / COMPOUND SHAPES AND PARTS OF SHAPES

Compound and partial shapes

Compound shape A shape formed by combining two or more simpler shapes. An L-shape can also be treated as a rectangle with a smaller rectangle removed.

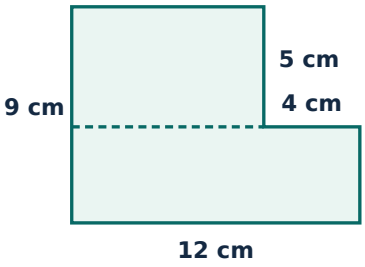
Part of a shape A portion of a familiar shape, such as a semicircle (half a circle) or a quadrant (one quarter of a circle).

Area and perimeter rules

Area = sum of the areas of the non-overlapping parts.
If a part is cut away, subtract its area from the original area.

Perimeter = sum of lengths along the boundary. A shared edge inside the shape is excluded. Include new edges around a cut-out.

Semicircle area = $\frac{1}{2}\pi r^2$ · Semicircle perimeter = $\pi r + 2r$
Quadrant area = $\frac{1}{4}\pi r^2$ · Quadrant perimeter = $\frac{1}{2}\pi r + 2r$



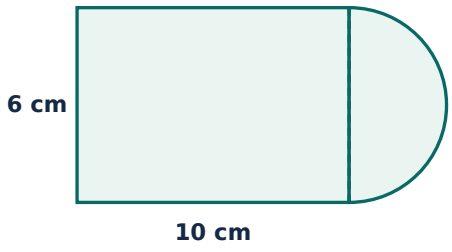
Worked example / L-shaped floor

A 12 cm by 9 cm rectangle has a 4 cm by 5 cm corner removed. Find the area and perimeter.

Area = $12 \times 9 - 4 \times 5 =$ **88 cm²**.

The remaining top edge is $12 - 4 = 8$ cm. The remaining right edge is $9 - 5 = 4$ cm.

Perimeter = $12 + 4 + 4 + 5 + 8 + 9 =$ **42 cm**.



Worked example / Rectangle and semicircle

A semicircle is attached to a 6 cm end of a 10 cm by 6 cm rectangle. Find the area and perimeter.

The semicircle has radius 3 cm.

Area = $10 \times 6 + \frac{1}{2}\pi \times 3^2 =$ **(60 + 4.5π) cm²**.

Perimeter = $10 + 10 + 6 + 3\pi =$ **(26 + 3π) cm**.

Worked example / A quadrant

Find the area and perimeter of a quadrant of radius 8 cm.

Area = $\frac{1}{4}\pi \times 8^2 =$ **16π cm²**. Perimeter = $\frac{1}{2}\pi \times 8 + 2 \times 8 =$ **(4π + 16) cm**.

E5.5 / COMPOUND SHAPES AND PARTS OF SHAPES

Compound shapes with arcs

Find the area by adding or subtracting parts. Trace the solid outline to find the perimeter. Dashed lines show joins or edges that were cut away.



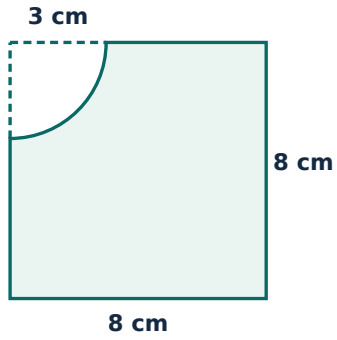
Worked example / Rectangle and quadrant

A quadrant of radius 4 cm is joined to the 4 cm side of a 9 cm by 4 cm rectangle. Find the area and perimeter.

Area = $9 \times 4 + \frac{1}{4}\pi \times 4^2$ = **$(36 + 4\pi)$ cm²**.

The shared 4 cm side is inside. The straight boundary is 9 + 9 + 4 + 4 = 26 cm.

Add the quarter-circle arc, $\frac{1}{2}\pi \times 4 = 2\pi$ cm.
Perimeter = **$(26 + 2\pi)$ cm**.



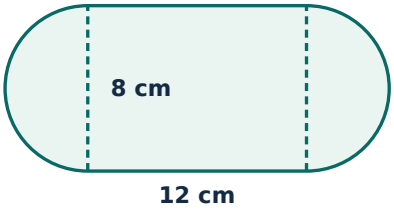
Worked example / Square with a quadrant cut out

A quadrant of radius 3 cm is cut from the corner of an 8 cm square. Find the area and perimeter.

Area = $8^2 - \frac{1}{4}\pi \times 3^2$ = **$(64 - 2.25\pi)$ cm²**.

The cut removes 3 cm from each of two sides. The straight boundary is $4 \times 8 - 2 \times 3 = 26$ cm.

The cut adds an arc of length $\frac{1}{2}\pi \times 3 = 1.5\pi$ cm.
Perimeter = **$(26 + 1.5\pi)$ cm**.



Worked example / Rectangle and two semicircles

A semicircle is attached to each 8 cm end of a 12 cm by 8 cm rectangle. Find the area and perimeter.

Each semicircle has radius 4 cm. Together they have the same area as a circle.

Area = $12 \times 8 + \pi \times 4^2$ = **$(96 + 16\pi)$ cm²**.

The two arcs make one full circumference. Add the two 12 cm sides. Perimeter = $24 + 2\pi \times 4$ = **$(24 + 8\pi)$ cm**.

E5.5 / COMPOUND SHAPES AND PARTS OF SHAPES

Compound and partial solids

Compound solid A solid made by joining two or more simpler solids.

Part of a solid A portion of a familiar solid. A hemisphere is half of a sphere, with a flat circular face when it is closed.

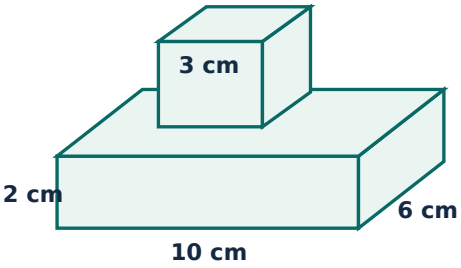
Cylindrical sector A prism with a uniform sector-shaped cross-section.

Volume and surface area rules

Volume Add the volumes of non-overlapping parts. If material is removed, subtract its volume.

Surface area Count every exposed surface. If you add the total areas of two joined solids, subtract twice the shared contact area. Cuts can expose new surfaces.

Hemisphere volume = $\frac{2}{3}\pi r^3$ · Curved surface area = $2\pi r^2$
Total surface area of a closed hemisphere = $3\pi r^2$



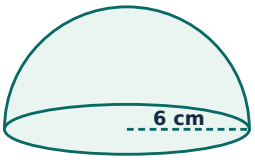
Worked example / Cube on a cuboid

A cube (six equal square faces) of side 3 cm sits on a 10 cm × 6 cm × 2 cm cuboid. Find the volume and surface area.

Volume = $10 \times 6 \times 2 + 3^3$ = **147 cm³**.

Cuboid surface area = $2(60 + 20 + 12) = 184$ cm².
Cube surface area = $6 \times 3^2 = 54$ cm².

The contact square is counted twice in the sum.
Surface area = $184 + 54 - 2(3 \times 3)$ = **220 cm²**.



Worked example / A closed hemisphere

Find the volume and total surface area when r = 6 cm.

Volume = $\frac{2}{3}\pi \times 6^3$ = **144π cm³**.

Curved area = 72π cm². Flat face area = 36π cm².

Total surface area = $72\pi + 36\pi$ = **108π cm²**.

Worked example / A quarter-cylinder

A quarter-cylinder has radius 4 cm and length 10 cm. Find its volume and total surface area.

End area = $\frac{1}{4}\pi \times 4^2 = 4\pi$ cm². Volume = $4\pi \times 10$ = **40π cm³**.

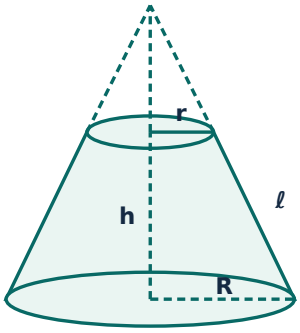
End perimeter = $4 + 4 + \frac{1}{4}(2\pi \times 4) = 8 + 2\pi$ cm.
Surface area = $2(4\pi) + 10(8 + 2\pi)$ = **(80 + 28π) cm²**.

E5.5 / COMPOUND SHAPES AND PARTS OF SHAPES

Frustums

Frustum The part of a cone or pyramid left when the tip is cut off by a plane parallel to the base.

Similar solids Solids with the same shape and proportional corresponding lengths. The original cone and the removed tip are similar, so their corresponding radii and heights have the same ratio.



Right circular cone frustum

Let R be the larger radius, r the smaller radius, h the perpendicular height and ℓ the slant height.

$\ell = \sqrt{[h^2 + (R - r)^2]}$
Volume $= \frac{1}{3}\pi h(R^2 + Rr + r^2)$
Curved surface area $= \pi(R + r)\ell$
Total surface area $= \pi(R + r)\ell + \pi R^2 + \pi r^2$

For a pyramid frustum, subtract the tip's volume from the original pyramid's volume. Find the exposed side areas and add the two end areas.

Worked example / Find volume by subtracting cones

A cone frustum has larger radius 6 cm, smaller radius 3 cm and perpendicular height 4 cm. Find its volume.

The removed cone has half the radius of the original cone, so it also has half the height. The original height is 8 cm and the tip height is 4 cm.

Original volume $= \frac{1}{3}\pi \times 6^2 \times 8 = 96\pi$ cm³.
Removed volume $= \frac{1}{3}\pi \times 3^2 \times 4 = 12\pi$ cm³.

Frustum volume $= 96\pi - 12\pi =$ **84π cm³**.

The frustum formula gives the same result.
$\frac{1}{3}\pi \times 4(6^2 + 6 \times 3 + 3^2) =$ **84π cm³**.

Worked example / Find the total surface area

Find the total surface area of the same frustum, including both circular ends.

Slant height $= \sqrt{[4^2 + (6 - 3)^2]} = 5$ cm.

Curved area $= \pi(6 + 3) \times 5 = 45\pi$ cm².
The two end areas total $\pi \times 6^2 + \pi \times 3^2 = 45\pi$ cm².

Total surface area $= 45\pi + 45\pi =$ **90π cm²**.

You can also find the curved area by subtracting the tip's curved area from the original cone's curved area. Then add the areas of the two exposed circular ends.

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