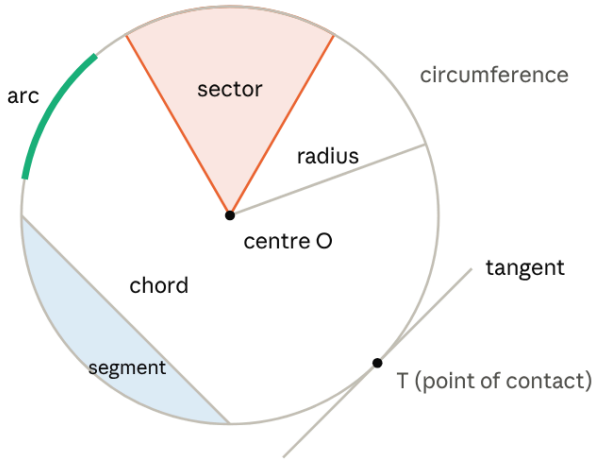
Circle Theorems — IGCSE 0580 (E4.7 & E4.8)

Key vocabulary

Learn these names first: every exam reason uses them.

Parts of a circle you must be able to name



Parts of a circle · labelled

Term	Meaning
Centre (O)	The point that every point on the circle is the same distance from
Radius	A line from the centre to the circumference; all radii of one circle are equal
Diameter	A chord through the centre; diameter = 2 × radius
Chord	A straight line joining two points on the circumference
Tangent	A straight line that touches the circle at exactly one point (the point of contact)

Term	Meaning
Arc	Part of the circumference; minor arc = shorter, major arc = longer
Segment	The region between a chord and its arc (minor or major segment)
Sector	The region between two radii and an arc
Cyclic quadrilateral	A four-sided shape with all four vertices on the circumference

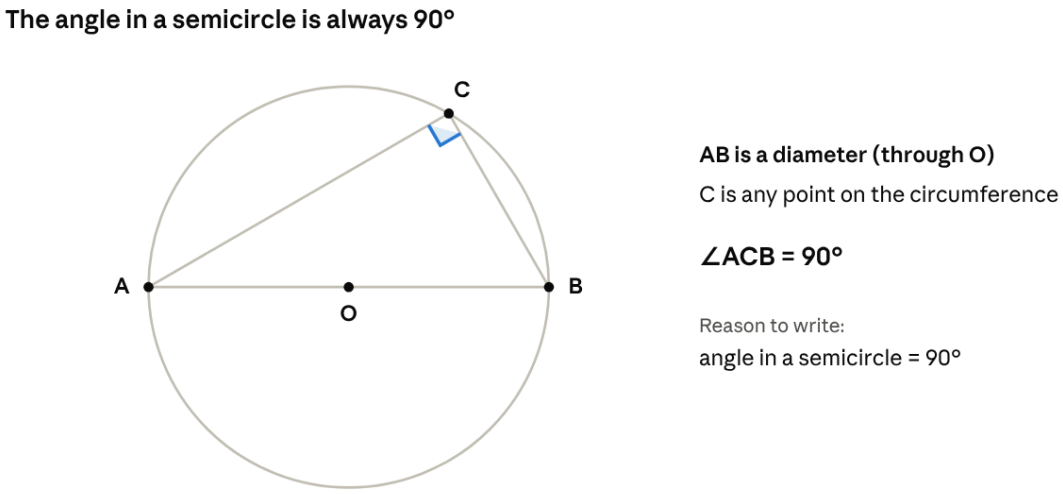
Subtended: an angle is subtended by a chord or arc when its two arms end at the ends of that chord or arc. In triangle ACB, ∠ACB is subtended by chord AB.

Always remember: two radii make an isosceles triangle (OA = OB), so its base angles are equal. Most circle questions need this alongside a theorem.

Part 1 — Circle theorems I (E4.7)

1. Angle in a semicircle = 90°

Any triangle drawn on a diameter, with its third corner on the circle, has a right angle at that corner.



Angle in a semicircle · AB is a diameter

Why it works: the angle at the centre (∠AOB) is a straight line, 180°. The angle at the circumference is half of it, so ∠ACB = 90°.

Spot it: a line through O with both ends on the circle, joined to a third point on the circle.

Exam reason: *angle in a semicircle = 90°*

Worked example

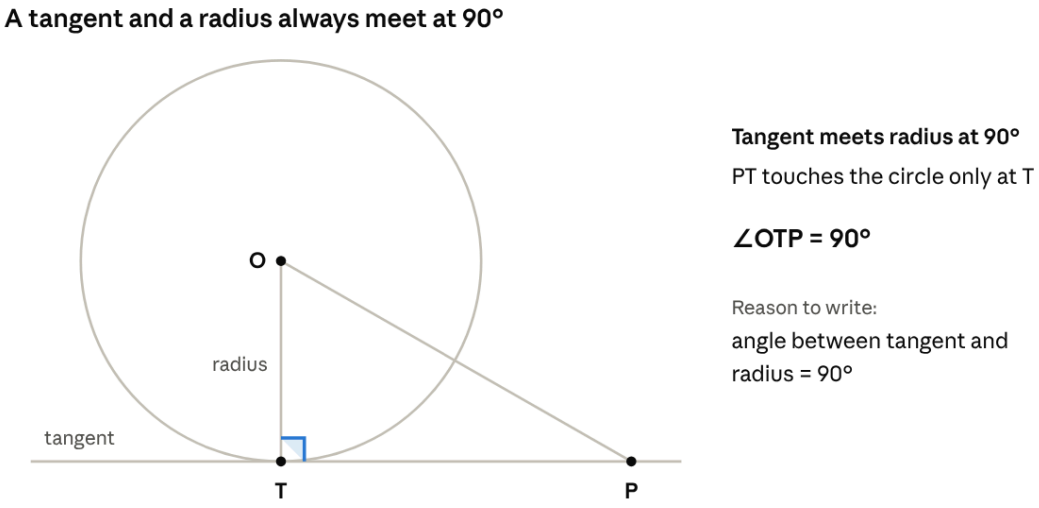
AB is a diameter of a circle with centre O. C is a point on the circumference and ∠CAB = 34°. Find ∠ABC, giving reasons.

1. ∠ACB = 90° (angle in a semicircle = 90°)
2. ∠ABC = 180° − 90° − 34° (angles in a triangle add up to 180°)

Answer: ∠ABC = 56°

2. Angle between tangent and radius = 90°

A tangent is always perpendicular to the radius drawn to its point of contact.



Tangent and radius · PT is a tangent at T

Spot it: a line touching the circle at one point, with the centre O joined to that point. Joining O to an outside point P then gives a right-angled triangle OTP, so Pythagoras and trigonometry often follow.

Exam reason: *angle between tangent and radius = 90°*

Worked example

PT is a tangent to the circle, centre O, at T. OT = 5 cm and PT = 12 cm.

(a) Find OP. (b) Find ∠TOP, correct to 1 decimal place.

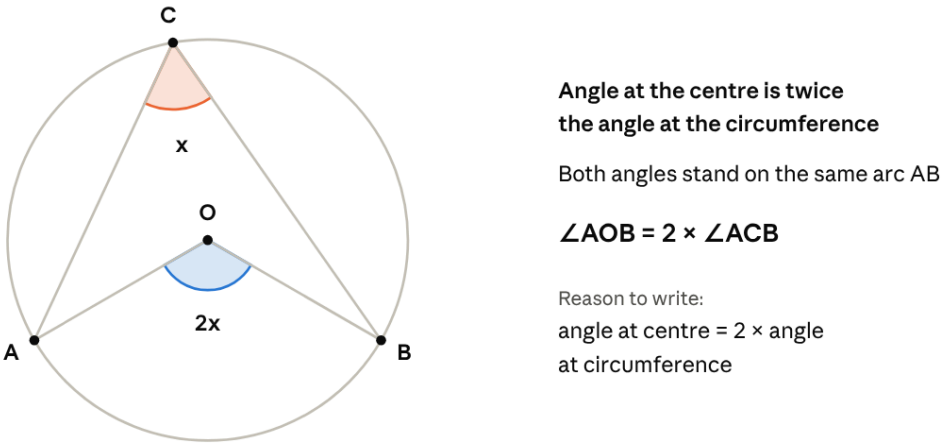
1. ∠OTP = 90° (angle between tangent and radius = 90°)
2. $OP^2 = 5^2 + 12^2 = 169$, so OP = 13 cm (Pythagoras)
3. tan ∠TOP = PT ÷ OT = 12 ÷ 5 = 2.4, so ∠TOP = $\tan^{-1}(2.4)$ = 67.4°

Answer: (a) OP = 13 cm (b) ∠TOP = 67.4°

3. Angle at the centre is twice the angle at the circumference

When two angles stand on the same arc, the one at the centre is double the one at the circumference.

The angle at the centre is twice the angle at the circumference



Angle at the centre · both angles stand on arc AB

Spot it: lines from the ends of an arc meet at O and also at a point on the circle. The shape often looks like an arrowhead, but it can also look like a kite or a bow.

Watch out: both angles must stand on the same arc, and the circumference point must be on the opposite side from that arc. It also works for a reflex angle at the centre.

Exam reason: *angle at centre = 2 × angle at circumference*

Worked example

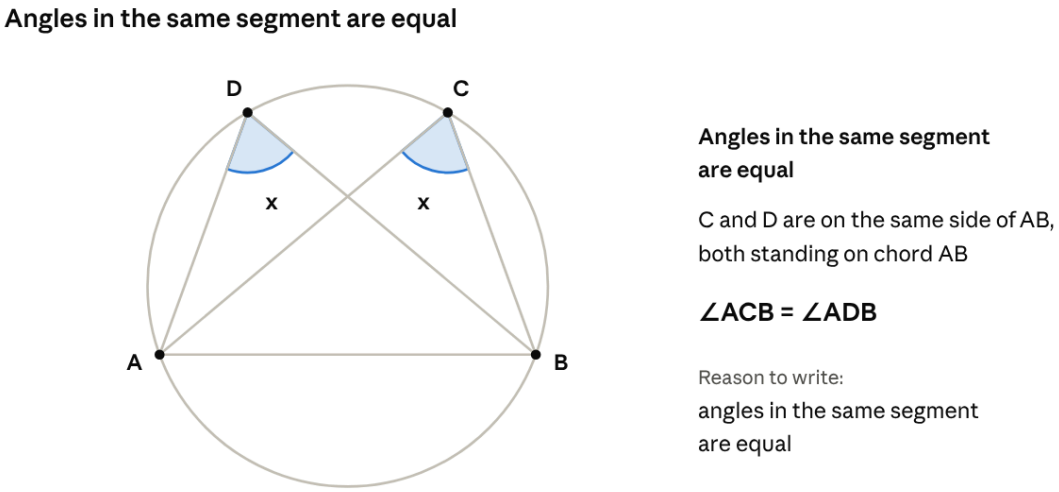
A, B and C lie on a circle with centre O. ∠AOB = 124°. Find ∠ACB and ∠OAB, giving reasons.

1. ∠ACB = 124° ÷ 2 = 62° (angle at centre = 2 × angle at circumference)
2. OA = OB (radii), so triangle OAB is isosceles
3. ∠OAB = (180° − 124°) ÷ 2 = 28° (base angles of an isosceles triangle are equal)

Answer: ∠ACB = 62°, ∠OAB = 28°

4. Angles in the same segment are equal

Angles at the circumference that stand on the same chord, on the same side of it, are equal.



Same segment · C and D both stand on chord AB

Why it works: ∠ACB and ∠ADB are each half of the same angle at the centre, ∠AOB.

Spot it: a bow-tie shape, with two crossing chords inside the circle. Look for pairs of angles at the circumference whose arms end at the same two points.

Exam reason: *angles in the same segment are equal*

Worked example

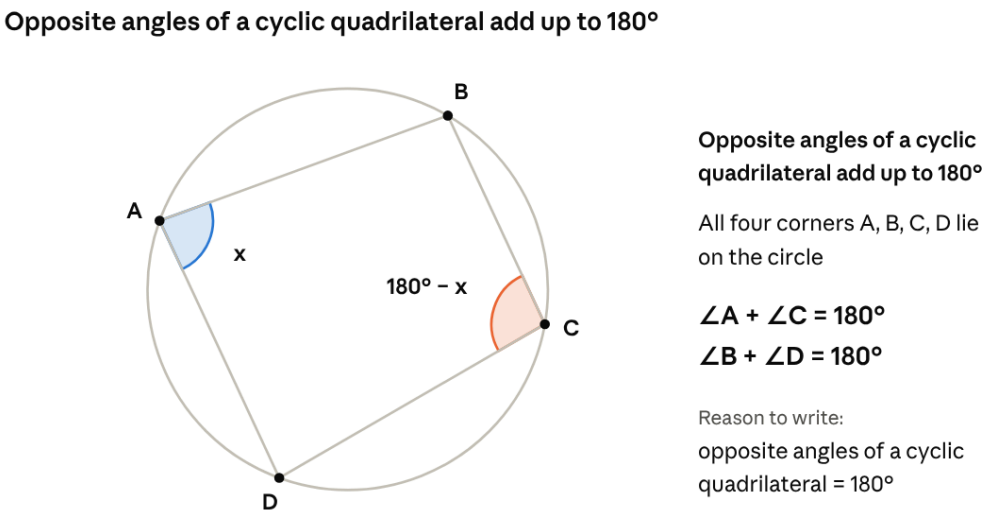
A, B, C and D lie on a circle (as in the diagram). Chords AC and BD cross. ∠ACB = 47° and ∠CBD = 31°. Find ∠ADB and ∠CAD, giving reasons.

1. ∠ADB and ∠ACB both stand on chord AB, so ∠ADB = 47° (angles in the same segment are equal)
2. ∠CAD and ∠CBD both stand on chord CD, so ∠CAD = 31° (angles in the same segment are equal)

Answer: ∠ADB = 47°, ∠CAD = 31°

5. Opposite angles of a cyclic quadrilateral sum to 180°

In a quadrilateral with all four corners on the circle, each pair of opposite angles is supplementary (adds up to 180°).



Cyclic quadrilateral ABCD · all corners on the circle

Why it works: ∠A and ∠C stand on the two arcs BD that make up the whole circle. Their centre angles add to 360°, so ∠A + ∠C = 180°.

Spot it: a four-sided shape whose corners are ALL on the circumference. If one corner is at the centre O, it is not cyclic.

Exam reason: *opposite angles of a cyclic quadrilateral = 180°*

Worked example

ABCD is a cyclic quadrilateral. ∠DAB = 108° and ∠ABC = 75°. Find ∠BCD and ∠ADC, giving reasons.

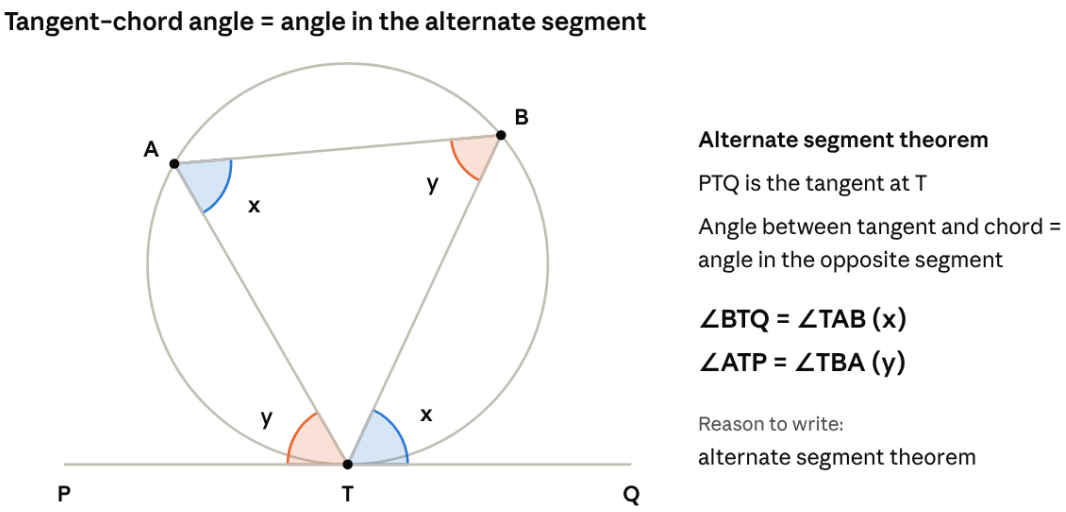
1. ∠BCD = 180° − 108° = 72° (opposite angles of a cyclic quadrilateral = 180°)

2. ∠ADC = 180° − 75° = 105° (opposite angles of a cyclic quadrilateral = 180°)
3. Check: 108° + 75° + 72° + 105° = 360° (angles in a quadrilateral)

Answer: ∠BCD = 72°, ∠ADC = 105°

6. Alternate segment theorem

The angle between a tangent and a chord equals the angle in the alternate (opposite) segment.



Alternate segment theorem · tangent PTQ at T

How to find the matching angle: start at the tangent–chord angle (e.g. ∠BTQ, between the tangent and chord TB). Go to the far end of that chord's triangle: the matching angle is at the third corner, A, standing on chord TB.

Spot it: a tangent and a triangle drawn inside the circle, with one corner of the triangle at the point of contact T.

Exam reason: *alternate segment theorem*

Worked example

PTQ is a tangent to the circle at T. A and B are on the circle. ∠BTQ = 58° and ∠ATP = 70°. Find ∠TAB, ∠TBA and ∠ATB, giving reasons.

1. ∠TAB = ∠BTQ = 58° (alternate segment theorem)
2. ∠TBA = ∠ATP = 70° (alternate segment theorem)
3. ∠ATB = 180° − 58° − 70° = 52° (angles on a straight line PTQ add up to 180°)

4. Check: 58° + 70° + 52° = 180° (angles in triangle TAB)

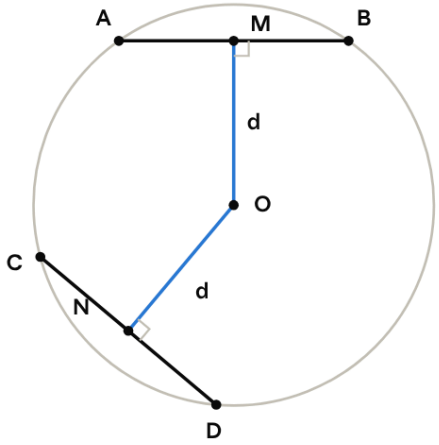
Answer: ∠TAB = 58°, ∠TBA = 70°, ∠ATB = 52°

Part 2 — Circle theorems II (E4.8)

7. Equal chords are equidistant from the centre

Chords of equal length are the same perpendicular distance from the centre. It works both ways: chords the same distance from O are equal.

Equal chords are the same distance from the centre



Equal chords are equidistant from the centre

If AB = CD, then OM = ON
OM and ON are the perpendicular distances from O to each chord

Reason to write:
equal chords are equidistant from the centre

Equal chords AB and CD · OM = ON

Key point: "distance from the centre" always means the perpendicular distance, OM, which meets the chord at 90° at its midpoint M. This makes right-angled triangles, so most questions need Pythagoras.

Exam reason: *equal chords are equidistant from the centre*

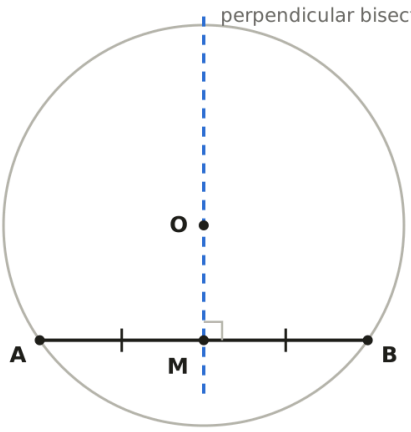
Worked example

A circle has centre O and radius 13 cm. Chords AB and CD are both 24 cm long. M and N are the midpoints of AB and CD. Find OM and ON.

1. OM is perpendicular to AB and bisects it, so AM = 24 ÷ 2 = 12 cm
2. OA = 13 cm (radius), so $OM^2 = 13^2 - 12^2 = 169 - 144 = 25$ (Pythagoras)
3. OM = 5 cm
4. AB = CD, so ON = OM = 5 cm (equal chords are equidistant from the centre)

8. The perpendicular bisector of a chord passes through the centre

The perpendicular bisector of any chord passes through O



The perpendicular bisector of a chord passes through the centre

M is the midpoint of chord AB
so AM = MB and ∠OMA = 90°

Reason to write:
perpendicular bisector of a chord passes through the centre

centre

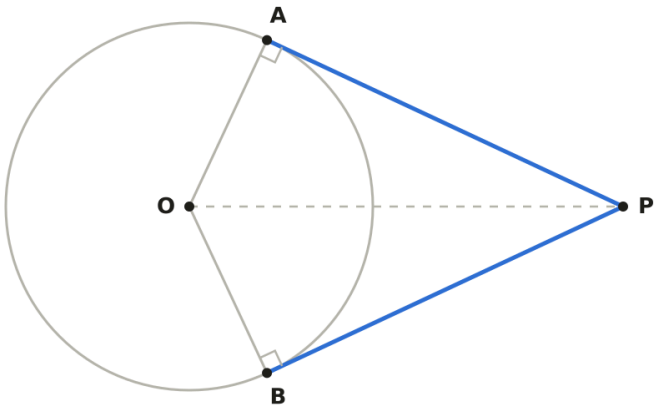
where they cross.

the perpendicular bisector of a chord passes through the centre (or *centre bisects the chord*)

1. OM is the perpendicular bisector of AB, so ∠OMA = 90° and AM = 16 ÷ 2 = 8 cm
2. $OA^2 = 8^2 + 6^2 = 64 + 36 = 1$
3. OA = 10 cm

9. Tangents from an external point are equal in length

Two tangents from the same outside point are equal ∠



Tangents from an external point are equal in length

PA and PB touch the circle at A and B

PA = PB

and ∠OAP = ∠OBP = 90°

Reason to write:
tangents from an external point are equal

it:

- ∠ ∠

- OAPB is a kite with right angles at A and B, so ∠AOB + ∠APB = 180°.
- OP is a line of symmetry: it bisects ∠APB and ∠AOB.

Exam reason: *tangents from an external point are equal*

Worked example

PA and PB are tangents from P to a circle with centre O, touching it at A and B. ∠APB = 48°. Find ∠PAB and ∠AOB, giving reasons.

1. PA = PB (tangents from an external point are equal), so triangle PAB is isosceles
2. ∠PAB = (180° − 48°) ÷ 2 = 66° (base angles of an isosceles triangle are equal)
3. ∠OAP = ∠OBP = 90° (angle between tangent and radius = 90°)
4. ∠AOB = 360° − 90° − 90° − 48° = 132° (angles in a quadrilateral add up to 360°)

Answer: ∠PAB = 66°, ∠AOB = 132°

Summary and exam tips

All nine theorems at a glance

#	Theorem	Look for	Reason to write
1	Angle in a semicircle	Triangle on a diameter	angle in a semicircle = 90°
2	Tangent and radius	Tangent with O joined to the point of contact	angle between tangent and radius = 90°
3	Angle at the centre	Arrowhead: lines from an arc to O and to the circle	angle at centre = 2 × angle at circumference
4	Same segment	Bow-tie: two angles standing on one chord	angles in the same segment are equal
5	Cyclic quadrilateral	Four corners all on the circle	opposite angles of a cyclic quadrilateral = 180°
6	Alternate segment	Tangent plus a triangle with a corner at the point of contact	alternate segment theorem

#	Theorem	Look for	Reason to write
7	Equal chords	Two chords of equal length, distances from O	equal chords are equidistant from the centre
8	Perpendicular bisector	Line from O at 90° to a chord	perpendicular bisector of a chord passes through the centre
9	Tangents from a point	Two tangents meeting outside the circle	tangents from an external point are equal

Exam tips

- **Give a reason for every angle.** In 0580 the reason often carries the mark, not just the number.
- **Use the syllabus wording** in the table above. Vague reasons such as "circle rule" or "because of the tangent" score nothing.
- **Look for radii first.** Two radii make an isosceles triangle, and most questions need this as well as a theorem.
- **Write each angle on the diagram as you find it.** The next step usually becomes obvious.
- **Don't assume.** A line is only a diameter or a tangent if the question says so, and diagrams are not drawn to scale. Use the given information, Never assume it from the diagram.
- **Combine with basic facts:** angles on a straight line (180°), around a point (360°), in a triangle (180°) and in a quadrilateral (360°).

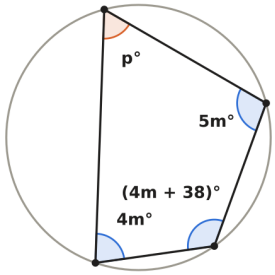
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Note: (Paper 2 became non-calculator from 2025)

Paper 2 — Extended, non-calculator (15 questions)

P2.1 · 0580/21/M/J/24 · Question 13 [3 marks]

Question (summary): A cyclic quadrilateral has angles p° at the top, 5m° on the right, (4m + 38)° at the bottom and 4m° on the left. Find p.

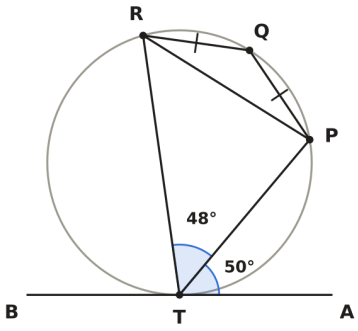


1 The left and right angles are opposite: 4m + 5m = 180 (opposite angles of a cyclic quadrilateral = 180°), so 9m = 180 and m = 20
2 Bottom angle = 4(20) + 38 = 118°
3 Top and bottom are opposite: p = 180 − 118 (opposite angles of a cyclic quadrilateral = 180°)

Answer: p = 62

P2.2 · 0580/23/M/J/24 · Question 14 [3 marks]

Question (summary): P, Q, R and T are on a circle. AB is a tangent at T. ∠ATP = 50°, ∠PTR = 48° and PQ = QR. (a) Find ∠PRT. (b) Find ∠QPR.

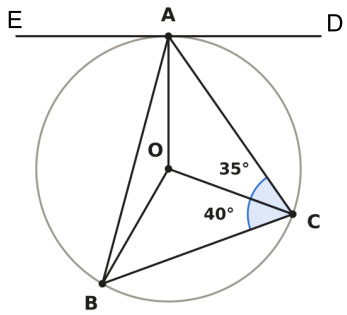


1 (a) ∠PRT = ∠ATP = 50° (alternate segment theorem)
2 (b) PQRT is a cyclic quadrilateral, so ∠PQR = 180° − 48° = 132° (opposite angles of a cyclic quadrilateral = 180°)
3 PQ = QR, so triangle PQR is isosceles: ∠QPR = (180° − 132°) ÷ 2 = 24°

Answer: (a) 50° (b) 24°

P2.3 · 0580/21/M/J/25 · Question 10 [4 marks]

Question (summary): A, B and C are on a circle, centre O. DE is a tangent at A. ∠ACO = 35° and ∠BCO = 40°. Find (a) ∠AOC (b) ∠ABC (c) ∠DAC (d) ∠OAB.



1 (a) OA = OC (radii), so ∠OAC = 35° and ∠AOC = 180° − 35° − 35° = 110°

2 (b) ∠ABC = 110° ÷ 2 = 55° (angle at centre = 2 × angle at circumference)

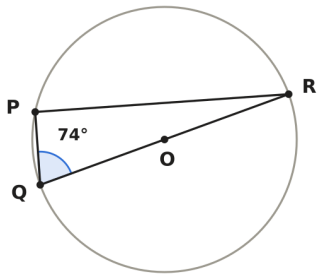
3 (c) ∠DAC = ∠ABC = 55° (alternate segment theorem). Check: ∠OAD = 90°, and 90° − 35° = 55°

4 (d) OB = OC (radii), so ∠OBC = 40°. Then ∠OBA = 55° − 40° = 15°, and ∠OAB = ∠OBA = 15° (OA = OB, isosceles triangle)

Answer: (a) 110° (b) 55° (c) 55° (d) 15°

P2.4 · 0580/22/M/J/25 · Question 12 [2 marks]

Question (summary): P, Q and R are on a circle and QR is a diameter. ∠PQR = 74°. Find ∠PRQ, giving reasons.



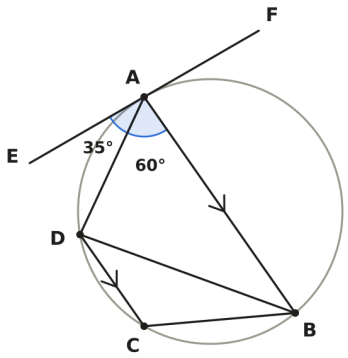
1 ∠QPR = 90° (angle in a semicircle = 90°)

2 ∠PRQ = 180° − 90° − 74° (angles in a triangle add up to 180°)

Answer: ∠PRQ = 16°

P2.5 · 0580/23/M/J/25 · Question 13 [4 marks]

Question (summary): A, B, C and D are on a circle. EF is a tangent at A and AB is parallel to DC. ∠EAD = 35° and ∠DAB = 60°. (a) Find ∠DCB, with a reason. (b) Find ∠DBC.



1 (a) ∠DCB = 180° − 60° = 120° (opposite angles of a cyclic quadrilateral = 180°)

2 (b) ∠ABD = ∠EAD = 35° (alternate segment theorem)

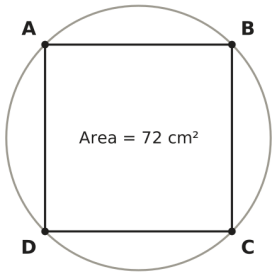
3 ∠BDC = ∠ABD = 35° (alternate angles, AB ∥ DC)

4 ∠DBC = 180° − 120° − 35° = 25° (angles in triangle BCD)

Answer: (a) 120° (b) 25°

P2.6 · 0580/22/O/N/24 · Question 9 [3 marks]

Question (summary): A, B, C and D are on a circle and ABCD is a square of area 72 cm². Find the area of the circle as a multiple of π.



1 ∠ABC = 90°, so diagonal AC is a diameter (angle in a semicircle = 90°) (Since ∠ABC is 90°, the chord AC opposite it is a diameter)

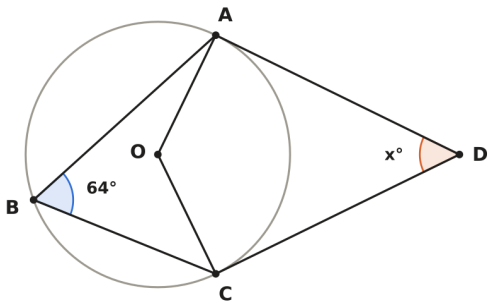
2 Side = √72 cm, so AC² = 72 + 72 = 144 and AC = 12 cm (Pythagoras)

3 Radius = 6 cm, so area = π × 6² = 36π

Answer: 36π cm²

P2.7 · 0580/22/O/N/24 · Question 17 [2 marks]

Question (summary): A, B and C are on a circle, centre O. DA and DC are tangents. ∠ABC = 64°. Find x, where x° = ∠ADC.



1 ∠AOC = 2 × 64° = 128° (angle at centre = 2 × angle at circumference)

2 ∠OAD = ∠OCD = 90° (angle between tangent and radius = 90°)

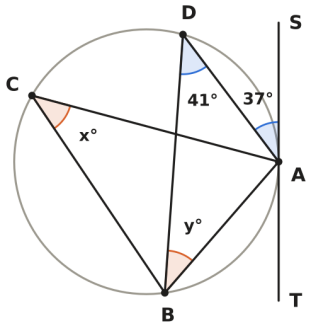
3 x = 360 − 90 − 90 − 128 (angles in quadrilateral OADC add up to 360°)

Answer: x = 52

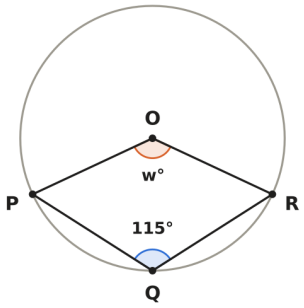
P2.8 · 0580/23/O/N/24 · Question 14 [4 marks]

Question (summary): (a) A, B, C and D are on a circle and TAS is a tangent at A. ∠SAD = 37° and ∠ADB = 41°. Find (i) x = ∠ACB and (ii) y = ∠ABD. (b) P, Q and R are on a circle, centre O, with Q on the minor arc PR and ∠PQR = 115°. Find w = ∠POR (the angle at O facing Q).

(a)



(b)



1 (a)(i) x = ∠ADB = 41 (angles in the same segment are equal)

2 (a)(ii) y = ∠SAD = 37 (alternate segment theorem)

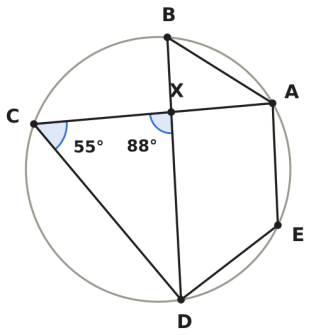
3 (b) An angle at the circumference on the major arc would be 180° − 115° = 65° (opposite angles of a cyclic quadrilateral)

4 w = 2 × 65 = 130 (angle at centre = 2 × angle at circumference)

Answer: (a)(i) x = 41 (ii) y = 37 (b) w = 130

P2.9 · 0580/21/O/N/25 · Question 14 [9 marks]

Question (summary): A, B, C, D and E are on a circle. AC and BD cross at X. ∠ACD = 55° and ∠CXD = 88°. (a) Find ∠CDB, ∠ABD and ∠AED with reasons. (b) Triangle CXD is similar to triangle BXA, with DX = 8.0 cm, BX = 2.7 cm and AX = 4.0 cm. (i) Find CX. (ii) Find the ratio of the areas of CXD to BXA.

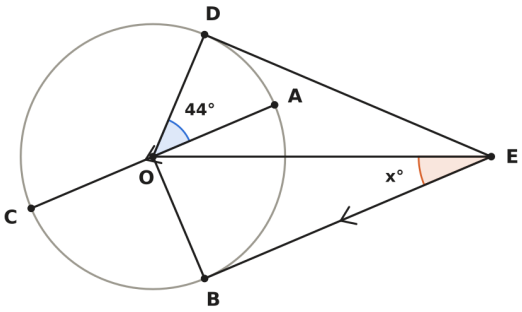


1 (a) ∠CDB = 180° − 55° − 88° = 37° (angles in a triangle add up to 180°)
2 ∠ABD = ∠ACD = 55° (angles in the same segment are equal)
3 ∠AED = 180° − 55° = 125° (opposite angles of cyclic quadrilateral ABDE = 180°)
4 (b)(i) Scale factor = DX ÷ AX = 8.0 ÷ 4.0 = 2, so CX = 2 × 2.7 = 5.4 cm
5 (b)(ii) Area ratio = $2^2 : 1 = 4 : 1$

Answer: (a) 37°, 55°, 125° (b)(i) 5.4 cm (ii) 4 : 1

P2.10 · 0580/23/O/N/25 · Question 13 [4 marks]

Question (summary): A, B, C and D are on a circle, centre O, with AC a diameter. ED and EB are tangents, AC is parallel to EB and ∠AOD = 44°. Find x = ∠OEB.

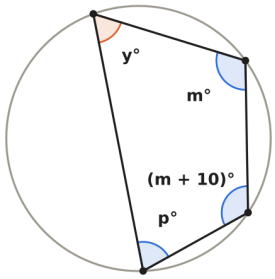


1 ∠OBE = 90° (angle between tangent and radius = 90°)
2 AC ∥ EB, so OB is also perpendicular to AC and ∠AOB = 90°
3 ∠DOB = 44° + 90° = 134°
4 OE is a line of symmetry of kite ODEB (tangents from an external point are equal), so ∠EOB = 134° ÷ 2 = 67°
5 x = 180 − 90 − 67 (angles in triangle OEB)

Answer: x = 23

P2.11 · 0580/23/O/N/25 · Question 21 [4 marks]

Question (summary): A cyclic quadrilateral has angles y° at the top, m° on the right, (m + 10)° at the bottom and p° on the left. p : m = 2 : 3. Find y.



1 p and m are opposite, so p + m = 180 (opposite angles of a cyclic quadrilateral = 180°)

2 p : m = 2 : 3, so m = 180 × 3/5 = 108

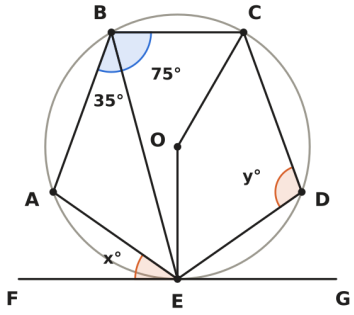
3 Bottom angle = 108 + 10 = 118°

4 y = 180 − 118 (opposite angles of a cyclic quadrilateral = 180°)

Answer: y = 62

P2.12 · 0580/22/F/M/26 · Question 16 [6 marks]

Question (summary): A, B, C, D and E are on a circle, centre O. FG is a tangent at E. ∠ABE = 35° and ∠CBE = 75°. x° is the angle between the tangent EF and chord EA. y° = ∠CDE. (a) Find x and y. (b) Give the reason why obtuse ∠EOC = 150°. (c) The radius is 6 cm. Find the area of the major sector EOC in terms of π.



1 (a) x = ∠ABE = 35 (alternate segment theorem)

2 B and D are on opposite sides of CE, so BCDE is a cyclic quadrilateral: y = 180 − 75 = 105

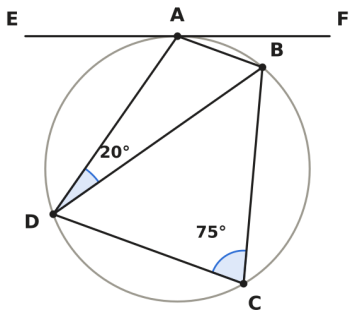
3 (b) The angle at the centre is twice the angle at the circumference: 2 × 75° = 150°

4 (c) Major sector angle = 360° − 150° = 210°, so area = 210/360 × π × 6² = 21π

Answer: (a) x = 35, y = 105 (b) angle at centre = 2 × angle at circumference (c) 21π cm²

P2.13 · 0580/22/M/J/26 · Question 15 [5 marks]

Question (summary): A, B, C and D are on a circle. EF is a tangent at A. ∠ADB = 20° and ∠BCD = 75°. Find ∠EAD, giving a reason for each step.



1 ∠DAB = 180° − 75° = 105° (opposite angles of a cyclic quadrilateral = 180°)

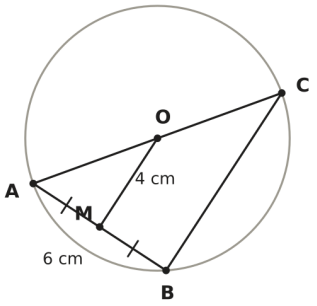
2 ∠ABD = 180° − 105° − 20° = 55° (angles in triangle ABD)

3 ∠EAD = ∠ABD = 55° (alternate segment theorem)

Answer: ∠EAD = 55°

P2.14 · 0580/23/M/J/26 · Question 13 [3 marks]

Question (summary): A, B and C are on a circle, centre O. AOC is a straight line and M is the midpoint of AB. AB = 6 cm and OM = 4 cm. (a) Find OA. (b) Give the reason why ∠ABC = 90°.



1 (a) OM joins the centre to the midpoint of chord AB, so OM ⊥ AB and AM = 3 cm (perpendicular bisector of a chord passes through the centre)

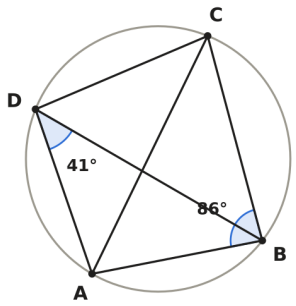
2 $OA^2 = 3^2 + 4^2 = 25$ (Pythagoras), so OA = 5 cm

3 (b) AC is a diameter, so ∠ABC = 90° (angle in a semicircle = 90°)

Answer: (a) 5 cm (b) angle in a semicircle = 90°

P2.15 · 0580/23/M/J/26 · Question 15 [3 marks]

Question (summary): ABCD is a cyclic quadrilateral with diagonals AC and BD. ∠ADB = 41° and ∠ABC = 86°. Find (a) ∠ACB (b) ∠BDC.



1 (a) ∠ACB = ∠ADB = 41° (angles in the same segment are equal)

2 (b) ∠ADC = 180° − 86° = 94° (opposite angles of a cyclic quadrilateral = 180°)

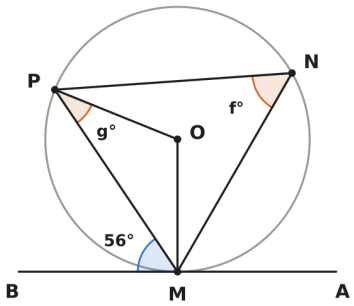
3 ∠BDC = 94° − 41° = 53°

Answer: (a) 41° (b) 53°

Paper 4 — Extended, calculator (7 questions)

P4.1 · 0580/42/M/J/24 · Question 2(c) [3 marks]

Question (summary): M, N and P are on a circle, centre O. AMB is a tangent at M and ∠PMB = 56°. f° = ∠MNP and g° = ∠OPM. Find f and g.



1 f = ∠PMB = 56 (alternate segment theorem)

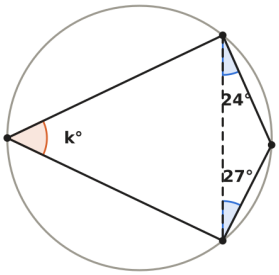
2 ∠OMB = 90° (angle between tangent and radius = 90°), so ∠OMP = 90° − 56° = 34°

3 OM = OP (radii), so g = ∠OMP = 34 (base angles of an isosceles triangle). Check: ∠MOP = 2 × 56° = 112°, and (180 − 112) ÷ 2 = 34

Answer: f = 56, g = 34

P4.2 · 0580/42/M/J/24 · Question 2(d) [2 marks]

Question (summary): A cyclic quadrilateral has a diagonal joining its top and bottom corners. At the top corner, the angle between the diagonal and the right-hand side is 24°. At the bottom corner it is 27°. k° is the angle at the left-hand corner. Find k.



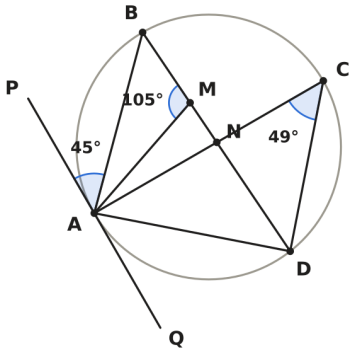
1 Right-hand corner = 180° − 24° − 27° = 129° (angles in a triangle add up to 180°)

2 k = 180 − 129 (opposite angles of a cyclic quadrilateral = 180°)

Answer: k = 51

P4.3 · 0580/42/O/N/24 · Question 4(b) [5 marks]

Question (summary): A, B, C and D are on a circle. PQ is the tangent at A. BMND is a straight line: M is a point on BD joined to A, and N is where AC crosses BD. ∠ACD = 49°, ∠AMB = 105° and ∠PAB = 45°. (i) Find ∠BAM. (ii)(a) Find ∠BAD. (ii)(b) Give a reason why BD is not a diameter.



1 (i) ∠ABD = ∠ACD = 49° (angles in the same segment are equal)

2 ∠BAM = 180° − 105° − 49° = 26° (angles in triangle ABM)

3 (ii)(a) ∠ADB = ∠PAB = 45° (alternate segment theorem)

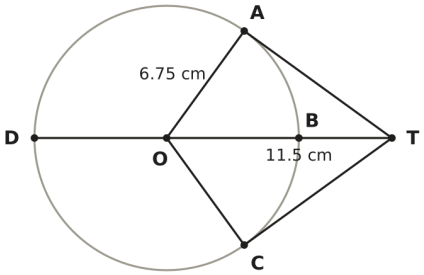
4 ∠BAD = 180° − 49° − 45° = 86° (angles in triangle ABD)

5 (ii)(b) If BD were a diameter, ∠BAD would be 90° (angle in a semicircle = 90°). It is 86°, so BD is not a diameter

Answer: (i) 26° (ii)(a) 86° (b) ∠BAD ≠ 90°, but the angle in a semicircle is 90°

P4.4 · 0580/42/O/N/24 · Question 4(c) [8 marks]

Question (summary): A, B, C and D are on a circle, centre O, where T, B, O and D lie on one straight line. TA and TC are tangents. OA = 6.75 cm and OT = 11.5 cm. (i) Show that ∠AOC = 108.12°, to 2 decimal places. (ii) Find the length of the minor arc ABC. (iii) Find the area of the major sector OCDA.

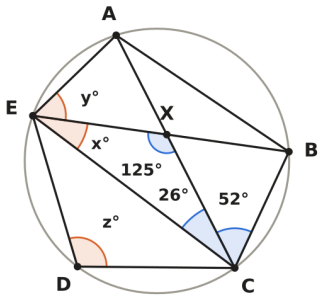


- (i) ∠OAT = 90° (angle between tangent and radius = 90°), so cos ∠AOT = 6.75 ÷ 11.5 and ∠AOT = 54.0587°
- Triangles OAT and OCT are congruent (tangents from an external point are equal), so ∠AOC = 2 × 54.0587° = 108.117...° = 108.12°
- (ii) Minor arc ABC = 108.12/360 × 2 × π × 6.75 = 12.7 cm
- (iii) Major sector angle = 360° − 108.12° = 251.88°, so area = 251.88/360 × π × 6.75² = 100 cm² (100.1)

Answer: (ii) 12.7 cm (iii) 100 cm²

P4.5 · 0580/42/M/J/25 · Question 18 [4 marks]

Question (summary): A, B, C, D and E are on a circle. Chords EB and AC cross, making a 125° angle in the triangle on the EC side. ∠ECA = 26° and ∠ACB = 52°. x° = ∠CEB, y° = ∠AEB and z° = ∠EDC. Find x, y and z.



1 x = 180 − 125 − 26 = 29 (angles in a triangle add up to 180°)

2 y = ∠ACB = 52 (angles in the same segment are equal)

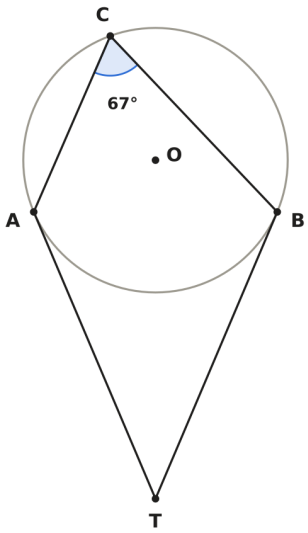
3 In triangle EBC: ∠ECB = 26° + 52° = 78°, so ∠EBC = 180° − 29° − 78° = 73°

4 z = 180 − 73 = 107 (opposite angles of cyclic quadrilateral EDCB = 180°)

Answer: x = 29, y = 52, z = 107

P4.6 · 0580/41/O/N/25 · Question 15 [3 marks]

Question (summary): A, B and C are on a circle, centre O. TA and TB are tangents at A and B. ∠BCA = 67°. Find ∠ATB.



1 ∠AOB = 2 × 67° = 134° (angle at centre = 2 × angle at circumference)

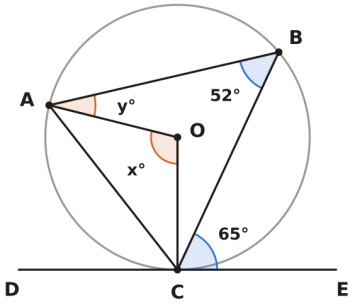
2 ∠OAT = ∠OBT = 90° (angle between tangent and radius = 90°)

3 ∠ATB = 360° − 90° − 90° − 134° (angles in quadrilateral OATB add up to 360°)

Answer: ∠ATB = 46°

P4.7 · 0580/42/O/N/25 · Question 12 [4 marks]

Question (summary): A, B and C are on a circle, centre O. DE is a tangent at C. ∠ABC = 52° and ∠BCE = 65°. x° = ∠AOC and y° = ∠BAO. (a) Find x, giving a reason. (b) Find y.



1 (a) x = 2 × 52 = 104 (angle at centre = 2 × angle at circumference)

2 (b) ∠BAC = ∠BCE = 65° (alternate segment theorem)

3 OA = OC (radii), so ∠OAC = (180° − 104°) ÷ 2 = 38°

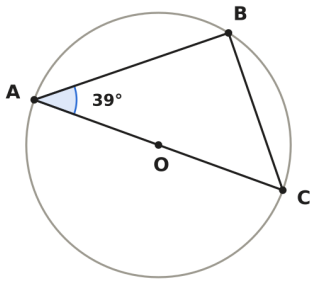
4 y = 65 − 38 = 27

Answer: (a) x = 104 (b) y = 27

Paper 1 — Core, non-calculator (4 questions)

P1.1 · 0580/11/M/J/25 · Question 21 [2 marks]

Question (summary): A, B and C are on a circle, centre O, and AC passes through O. ∠BAC = 39°. Find ∠BCA.

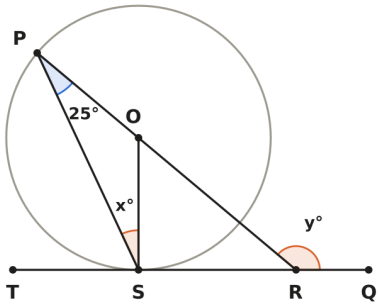


1 AC is a diameter, so ∠ABC = 90° (angle in a semicircle = 90°)
2 ∠BCA = 180° − 90° − 39° (angles in a triangle add up to 180°)

Answer: ∠BCA = 51°

P1.2 · 0580/13/M/J/25 · Question 18 [5 marks]

Question (summary): P and S are on a circle, centre O. POR is a straight line and QRST is a tangent at S. ∠OPS = 25°. x° = ∠OSP and y° = ∠QRO. (a) Find x, with a reason. (b) Find y.

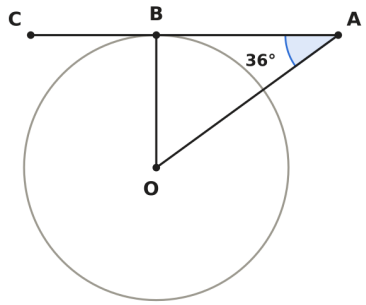


1 (a) OP = OS (radii), so triangle OPS is isosceles and x = 25 (base angles of an isosceles triangle are equal)
2 (b) ∠POS = 180° − 25° − 25° = 130°, so ∠SOR = 180° − 130° = 50° (angles on a straight line)
3 ∠OSR = 90° (angle between tangent and radius = 90°), so ∠ORS = 180° − 90° − 50° = 40°
4 y = 180 − 40 (angles on a straight line QRS)

Answer: (a) x = 25 (b) y = 140

P1.3 · 0580/12/O/N/25 · Question 24 [1 mark]

Question (summary): B is on a circle, centre O. ABC is a tangent at B and ∠OAB = 36°. Find ∠AOB.



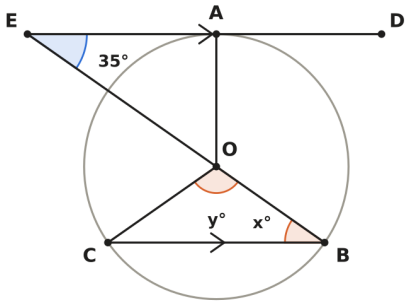
1 ∠OBA = 90° (angle between tangent and radius = 90°)

2 ∠AOB = 180° − 90° − 36° (angles in a triangle add up to 180°)

Answer: ∠AOB = 54°

P1.4 · 0580/13/M/J/26 · Question 9 [5 marks]

Question (summary): A, B and C are on a circle, centre O. DAE is a tangent at A, DE is parallel to BC and BOE is a straight line. ∠AEO = 35°. (a) Give the two reasons why ∠AOE = 55°. (b) Find x = ∠OBC. (c) Find y = ∠BOC.



1 (a) ∠OAE = 90° (angle between tangent and radius = 90°), then ∠AOE = 180° − 90° − 35° = 55° (angles in a triangle add up to 180°)

2 (b) x = ∠BED = 35 (alternate angles, DE ∥ BC)

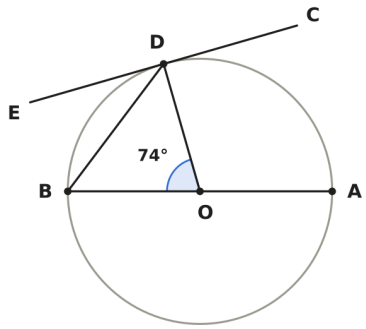
3 (c) OB = OC (radii), so ∠OCB = 35° and y = 180 − 35 − 35 = 110

Answer: (a) tangent ⊥ radius; angles in a triangle = 180° (b) x = 35 (c) y = 110

Paper 3 — Core, calculator (9 questions)

P3.1 · 0580/32/M/J/24 · Question 5(c) [5 marks]

Question (summary): A circle has centre O and diameter AOB. The line CDE touches the circle at D, and ∠DOB = 74°. (i) Name the line CDE. (ii) Find ∠ODB. (iii) Find ∠BDE, with a reason.



1 (i) CDE is a tangent

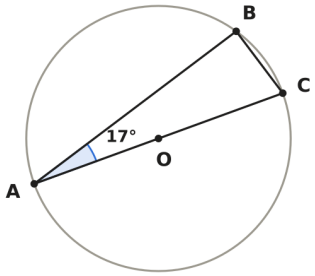
2 (ii) OD = OB (radii), so ∠ODB = (180° − 74°) ÷ 2 = 53°

3 (iii) ∠ODE = 90° (angle between tangent and radius = 90°), so ∠BDE = 90° − 53° = 37°

Answer: (i) tangent (ii) 53° (iii) 37°

P3.2 · 0580/33/M/J/24 · Question 5(d) [2 marks]

Question (summary): A, B and C are on a circle, centre O, and AC passes through O. ∠BAC = 17°. Find ∠ACB.



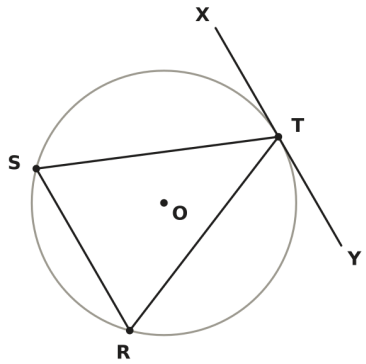
1 ∠ABC = 90° (angle in a semicircle = 90°)

2 ∠ACB = 180° − 90° − 17°

Answer: ∠ACB = 73°

P3.3 · 0580/31/O/N/24 · Question 2(b) [3 marks]

Question (summary): R, S and T are on a circle, centre O. Line XY touches the circle at T. (i) Name the line XY. (ii) Name the line SR. (iii) Toby thinks RST is a right-angled triangle; O is not on any side of the triangle. Give a reason why he is wrong.



1 (i) XY is a tangent

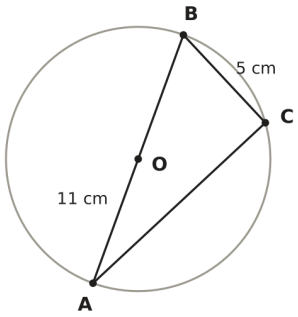
2 (ii) SR is a chord

3 (iii) No side of RST is a diameter, so none of its angles is an angle in a semicircle and none has to be 90°

Answer: (i) tangent (ii) chord (iii) none of the sides is a diameter

P3.4 · 0580/33/O/N/24 · Question 7(c) [6 marks]

Question (summary): A, B and C are on a circle, centre O, and AB passes through O. AB = 11 cm and BC = 5 cm. (i) Give a reason why ∠ACB = 90°. (ii) Find the circumference of the circle. (iii) Show that AC = 9.8 cm to 2 significant figures.



1 (i) AB is a diameter, so ∠ACB = 90° (angle in a semicircle = 90°)

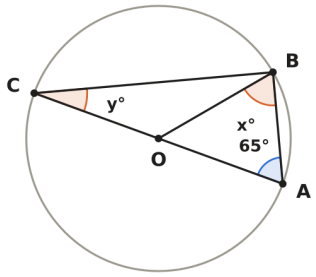
2 (ii) Circumference = π × 11 = 34.6 cm

3 (iii) $AC^2 = 11^2 - 5^2 = 121 - 25 = 96$ (Pythagoras), so AC = 9.797… = 9.8 cm

Answer: (ii) 34.6 cm (iii) AC = 9.8 cm

P3.5 · 0580/32/M/J/25 · Question 10 [5 marks]

Question (summary): A, B and C are on a circle, centre O, with diameter AC. ∠BAO = 65°, x° = ∠ABO and y° = ∠BCA. Give reasons for: (a) the value of x (b) ∠ABC = 90° (c) the value of y.



1 (a) x = 65, because OA = OB (radii) and base angles of an isosceles triangle are equal

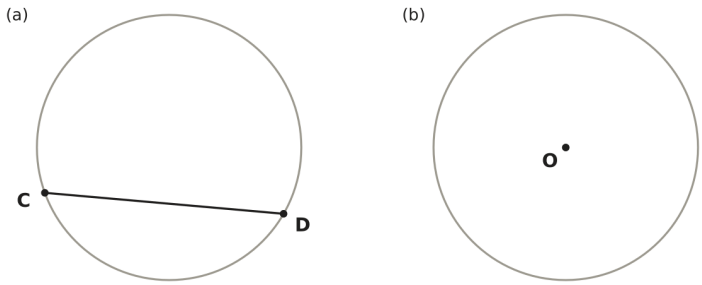
2 (b) ∠ABC = 90° because the angle in a semicircle = 90°

3 (c) y = 180 − 90 − 65 = 25, because angles in a triangle add up to 180°

Answer: (a) 65 (c) 25

P3.6 · 0580/33/M/J/25 · Question 18 [2 marks]

Question (summary): (a) C and D are on a circle. Name the line CD. (b) A circle has centre O. Draw triangle KLM with K, L and M on the circumference so that ∠KLM = 90°.



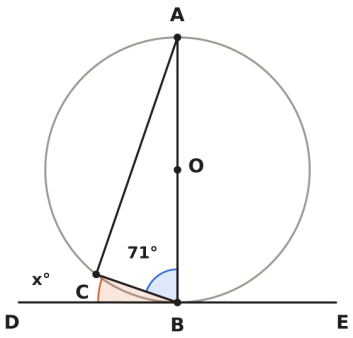
1 (a) CD is a chord

2 (b) Draw KM as a diameter (a straight line through O), then put L anywhere else on the circle. ∠KLM = 90° (angle in a semicircle = 90°)

Answer: (a) chord (b) any triangle with KM through O

P3.7 · 0580/31/O/N/25 · Question 20(b) [2 marks]

Question (summary): A, B and C are on a circle, centre O, and AB is a diameter. DBE is a tangent at B. ∠CBA = 71° and x° = ∠DBC. Find x, with a reason.



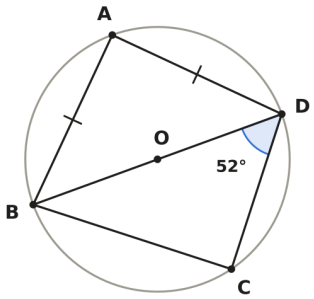
1 ∠DBA = 90° (angle between tangent and radius = 90°)

2 x = 90 − 71

Answer: x = 19

P3.8 · 0580/31/M/J/26 · Question 19 [3 marks]

Question (summary): A, B, C and D are on a circle, centre O. BD is a diameter, AB = AD and ∠BDC = 52°. Find ∠ABC.



1 ∠BAD = 90° (angle in a semicircle), and AB = AD, so ∠ABD = (180° − 90°) ÷ 2 = 45°

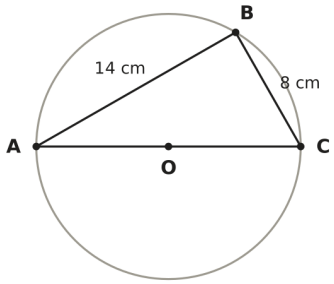
2 ∠BCD = 90° (angle in a semicircle), so ∠DBC = 180° − 90° − 52° = 38°

3 ∠ABC = 45° + 38°

Answer: ∠ABC = 83°

P3.9 · 0580/33/M/J/26 · Question 25 [5 marks]

Question (summary): A circle has centre O and diameter AC. B is on the circumference, with AB = 14 cm and BC = 8 cm. (a) Give the reason why ∠ABC = 90°. (b) Find AC. (c) Find ∠BAC.



1 (a) Angle in a semicircle = 90°

2 (b) $AC^2 = 14^2 + 8^2 = 260$ (Pythagoras), so AC = 16.1 cm

3 (c) $\tan \angle BAC = 8 \div 14$, so $\angle BAC = \tan^{-1}(0.5714) = 29.7°$

Answer: (b) 16.1 cm (c) 29.7°

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