E4.6 / ANGLES

Angle language and notation

An **angle** measures the amount of turn between two rays meeting at a point. A **degree** (°) is one 360th of a full turn. One full turn is 360°. Diagrams throughout these notes are not drawn to scale.

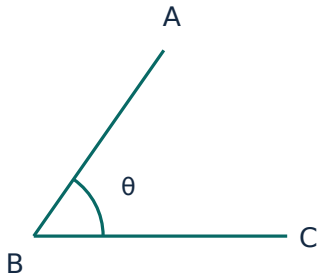
Point An exact position, usually labelled with a capital letter.

Line and line segment A line extends indefinitely in both directions. A segment is the part between two endpoints.

Ray A part of a line starting at one endpoint and extending in one direction.

Vertex The point where two rays or sides meet. The plural is vertices.

Arms of an angle The two rays forming the angle.



Three-letter angle notation

∠ABC means the angle formed by BA and BC. **The middle letter B is the vertex.** ∠CBA names the same angle.

θ (theta) is a letter often used to represent an angle. In a labelled calculation, x or y can also represent an unknown angle.

Types of angle

Acute $0° < \theta < 90°$ · **Right** $\theta = 90°$

Obtuse $90° < \theta < 180°$ · **Straight** $\theta = 180°$

Reflex $180° < \theta < 360°$ · **Full turn** $\theta = 360°$

Worked example / Classifying angles

Classify 37°, 90°, 136°, 180° and 245°.

37° is acute. 90° is right. 136° is obtuse. 180° is straight. 245° is reflex.

E4.6 / ANGLES

Basic angle rules

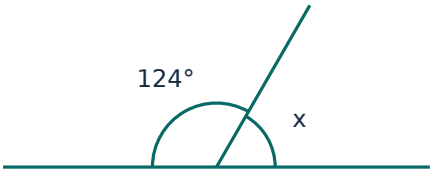
Adjacent angles Angles sharing a vertex and one arm, with no overlap in their interiors.

Complementary angles Two angles with a sum of 90°.

Supplementary angles Two angles with a sum of 180°.

Perpendicular lines Lines that meet at a right angle. The symbol ⊥ means perpendicular.

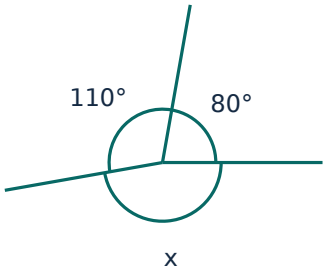
Intersecting lines Lines that meet. The angles directly opposite each other at the intersection are vertically opposite angles.



Angles on a straight line

Adjacent angles forming a straight angle add to **180°**.

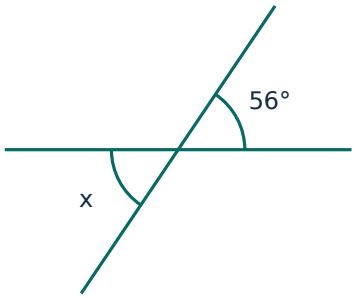
Worked example x + 124° = 180°, so **x = 56°**.



Angles at a point

All the angles around a point add to **360°**.

Worked example x + 80° + 110° = 360°, so **x = 170°**.



Vertically opposite angles

Vertically opposite angles are **equal**.

Worked example x = 56°, because vertically opposite angles are equal.

State the rule you used. For example, “angles on a straight line sum to 180°” gives a clear geometrical reason.

E4.6 / ANGLES

Triangles and quadrilaterals

Triangle A polygon with three sides. A polygon is a closed flat shape made of straight line segments.

Isosceles triangle A triangle with two equal sides. The angles opposite those sides are equal.

Equilateral triangle A triangle with three equal sides and three angles of 60°.

Scalene triangle A triangle with no equal sides.

Right-angled triangle A triangle with one angle of 90°. Its other two angles add to 90°.

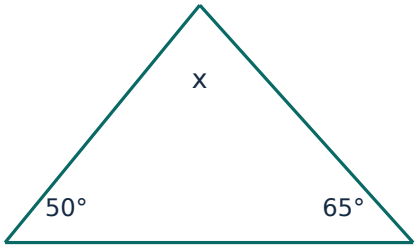
Quadrilateral A polygon with four sides. A diagonal joins two vertices that are not next to each other.

Angle sums

Triangle $a + b + c = \mathbf{180°}$

Quadrilateral $a + b + c + d = \mathbf{360°}$

A quadrilateral can be split into two triangles by a diagonal, giving $2 \times 180° = 360°$.



Worked example / Triangle

Two angles are 50° and 65°. Find x.

$x = 180° - 50° - 65° = \mathbf{65°}$.

Reason Angles in a triangle sum to 180°.

Worked example / Isosceles triangle

An isosceles triangle has vertex angle 44°. Find each base angle.

The base angles are equal. Each is $(180° - 44°) \div 2 = \mathbf{68°}$.

Worked example / Quadrilateral

A quadrilateral has angles 78°, 92°, 105° and x.

$x = 360° - (78° + 92° + 105°) = \mathbf{85°}$.

E4.6 / ANGLES

Angles in parallel lines

Parallel lines Lines in the same plane that remain the same distance apart and never meet. The symbol ∥ means parallel. Matching arrow marks identify parallel lines.

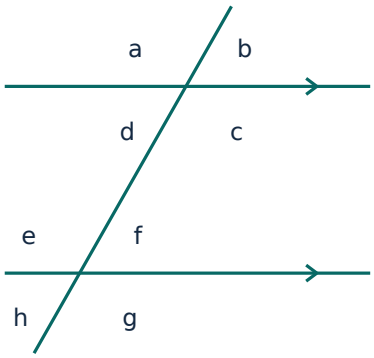
Transversal A line crossing two or more other lines.

Interior angles Angles in the region between the two lines. Exterior angles are outside that region.

Corresponding angles Angles in matching positions at the two intersections.

Alternate angles Angles on opposite sides of the transversal, both inside or both outside the two lines.

Co-interior angles Interior angles on the same side of the transversal.



Rules when the two lines are parallel

Corresponding angles are equal.
a = e, b = f, c = g, d = h.

Alternate angles are equal.
Interior pairs d = f and c = e.
Exterior pairs a = g and b = h.

Co-interior angles sum to 180°.
d + e = 180° and c + f = 180°.

Worked example / Using the labelled diagram

In the diagram above, b = 68°. Find f, d and e.

f = 68°, because corresponding angles are equal.

d = 68°, because vertically opposite angles are equal.

e = 112°, because d + e = 180° (co-interior angles).

Worked example / Algebra with parallel lines

Two co-interior angles are (3x + 12)° and (2x + 18)°. Find x and both angles.

3x + 12 + 2x + 18 = 180, so 5x + 30 = 180 and **x = 30**.

The angles are 3(30) + 12 = **102°** and 2(30) + 18 = **78°**.

E4.6 / ANGLES

Interior and exterior angles

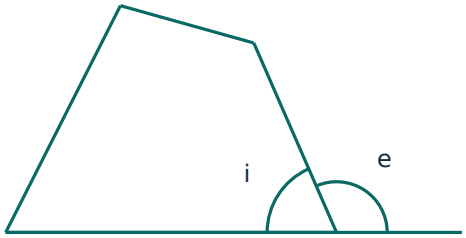
Interior angle The angle inside a polygon at a vertex.

Exterior angle The angle between a side and the extension of an adjacent side. For a convex polygon it is supplementary to the interior angle.

Convex polygon A polygon with every interior angle less than 180°. A concave polygon has at least one reflex interior angle.

Regular polygon A polygon with all sides equal and all interior angles equal.

Irregular polygon A polygon whose sides or interior angles are not all equal.



Interior + adjacent exterior = 180°

i + e = 180°

For a convex polygon, the exterior angles, one at each vertex taken in the same direction, sum to 360°.

Polygon formulas / n is the number of sides

Sum of interior angles = **(n − 2) × 180°**

For a regular polygon, each interior angle = **[(n − 2) × 180°] ÷ n**.

For a regular polygon, each exterior angle = **360° ÷ n**.

For a regular polygon, n = **360° ÷ one exterior angle**.

An irregular polygon still has the same interior angle sum. Its individual angles can have different sizes.

Worked examples / A regular hexagon

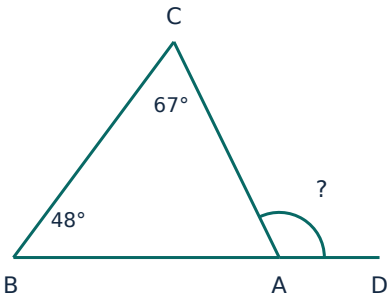
A hexagon has six sides. Find its interior angle sum and, when regular, each interior and exterior angle.

Sum = (6 − 2) × 180° = **720°**. Each interior angle = 720° ÷ 6 = **120°**.

Each exterior angle = 360° ÷ 6 = **60°**. Check 120° + 60° = 180°.

E4.6 / ANGLES

Worked problems



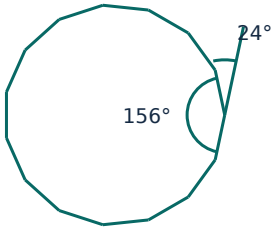
1 / Triangle with an extended side

In triangle ABC, ∠ABC = 48° and ∠BCA = 67°. Side BA is extended beyond A to D. Find ∠CAD.

∠BAC = 180° − 48° − 67° = 65° (angles in a triangle).

∠CAD = 180° − 65° = **115°** (angles on a straight line).

This also gives the triangle exterior-angle rule. An exterior angle equals the sum of the two opposite interior angles, so 48° + 67° = 115°.



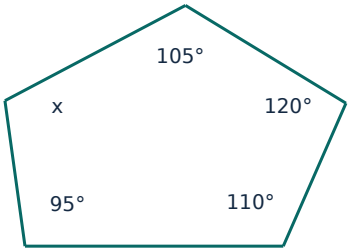
Regular polygon

2 / Number of sides from an interior angle

Each interior angle of a regular polygon is 156°. Find the number of sides.

Each exterior angle = 180° − 156° = 24°.

n = 360° ÷ 24° = **15 sides** (exterior angles sum to 360°).

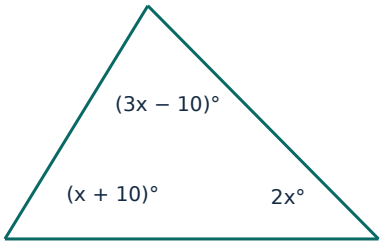


3 / An irregular pentagon

A pentagon has interior angles 95°, 110°, 120°, 105° and x. Find x.

A pentagon has 5 sides, so the angle sum is (5 − 2) × 180° = 540°.

x = 540° − (95° + 110° + 120° + 105°) = **110°**.



4 / Algebra in a triangle

A triangle has angles (x + 10)°, 2x° and (3x − 10)°. Find all three angles.

(x + 10) + 2x + (3x − 10) = 180 (triangle angle sum).

6x = 180, so x = 30. The angles are **40°, 60° and 80°**.

Useful polygon names

Triangle 3 sides · Quadrilateral 4 · Pentagon 5
Hexagon 6 · Heptagon 7 · Octagon 8
Nonagon 9 · Decagon 10 · Dodecagon 12

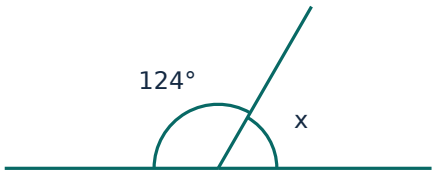
When explaining an answer, name the angle rule and identify the relevant angles. Keep exact values until the final step.

EXAM STYLE PRACTICE / 1 OF 3

Basic angle questions

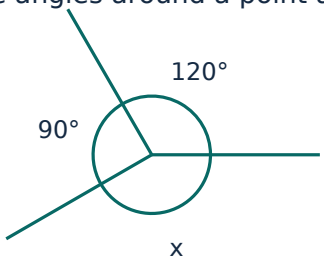
Original practice questions based on E4.6. Suggested marks are shown in brackets. Diagrams are not drawn to scale. Give a geometrical reason where requested.

1. Two adjacent angles on a straight line are 124° and x. Find x. **[2]**



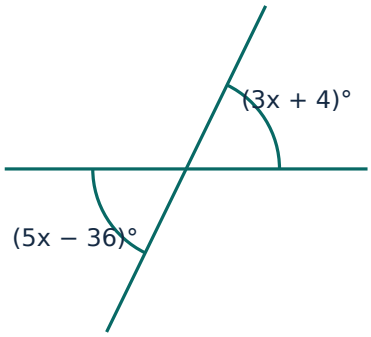
Show your working and give a geometrical reason.

2. The angles around a point are 120°, 90° and x. Find x. **[2]**



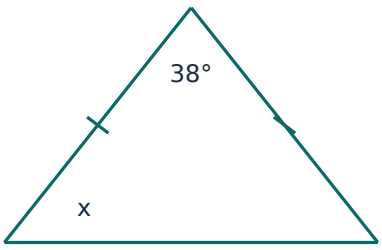
Show your working and give a geometrical reason.

3. The marked angles are vertically opposite. Find x. **[2]**



Show your working and give a geometrical reason.

4. The two marked sides of the triangle are equal. Find the base angle x. **[3]**



Show your working and give a geometrical reason.

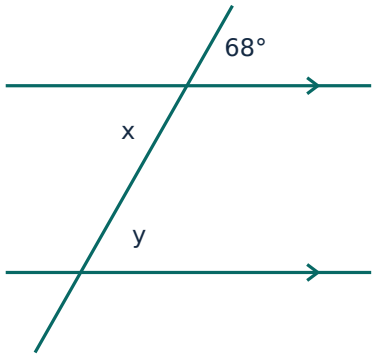
EXAM STYLE PRACTICE / 2 OF 3

Parallel lines and reasoning

Use the arrow marks to identify parallel lines. Show working and give the reasons needed for each answer. Diagrams are not drawn to scale.

5. A quadrilateral has interior angles 83°, 97°, 112° and x. Find x and give a reason. **[2]**

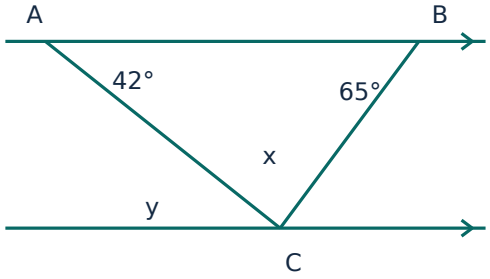
6. The two horizontal lines are parallel. Find x and y, giving a reason for each. **[3]**



Show your working and give a geometrical reason.

7. Two co-interior angles between parallel lines are (4x + 5)° and (2x + 25)°. Find x and both angles. **[3]**

8. The horizontal lines are parallel. The two interior angles at A and B are 42° and 65°. Find x = ∠ACB and the marked angle y. Give reasons. **[4]**



Show your working and give a geometrical reason.

EXAM STYLE PRACTICE / 3 OF 3

Polygon problems

Use angle sums and exterior angles to solve these questions. Show your working. There are 34 suggested marks across Questions 1 to 12.

9. A heptagon has six interior angles of 120°, 125°, 130°, 135°, 140° and 110°. Calculate the seventh interior angle. **[3]**

10. Each interior angle of a regular polygon is 162°. Calculate its exterior angle and the number of sides. **[3]**

11. The four interior angles of a quadrilateral are in the ratio 2 : 3 : 4 : 6. Find all four angles. **[3]**

12. A regular pentagon and a regular hexagon share a side. At one endpoint of that side, their interiors lie on opposite sides of the shared side. Find the remaining angle around that point. Show how you find both polygon angles. **[4]**

WORKED ANSWERS / 1 OF 2

Answers 1 to 6

The marks below are suggested for this original practice set. Award equivalent valid working and geometrical reasons.

1 / Straight line [2]

$x = 180° - 124° =$ **56°**.
1 mark for 56°. 1 mark for stating that angles on a straight line sum to 180°.

2 / Angles at a point [2]

$x = 360° - 120° - 90° =$ **150°**.
1 mark for 150°. 1 mark for stating that angles around a point sum to 360°.

3 / Vertically opposite angles [2]

$3x + 4 = 5x - 36$ because vertically opposite angles are equal.
$40 = 2x$, so **$x = 20$**. Each marked angle is 64°.
1 mark for the equation or equivalent working. 1 mark for $x = 20$.

4 / Isosceles triangle [3]

The base angles are equal because the two opposite sides are equal.
$2x + 38° = 180°$, so $x = 142° \div 2 =$ **71°**.
1 mark for equal base angles. 1 mark for using the 180° triangle sum. 1 mark for 71°.

5 / Quadrilateral [2]

$x = 360° - (83° + 97° + 112°) =$ **68°**.
1 mark for 68°. 1 mark for stating that quadrilateral angles sum to 360°.

6 / Parallel lines [3]

$x = 112°$, because x and 68° form a straight angle.
$y = 68°$, because corresponding angles in parallel lines are equal.
1 mark for each value. 1 mark for both correct reasons.

WORKED ANSWERS / 2 OF 2

Answers 7 to 12

7 / Co-interior angles [3]

(4x + 5) + (2x + 25) = 180 because co-interior angles sum to 180°.
6x + 30 = 180, so **x = 25**. The angles are **105° and 75°**.
1 mark for the equation. 1 mark for x = 25. 1 mark for both angles.

8 / Triangle between parallel lines [4]

x = 73°, since 180° − 42° − 65° = 73° (triangle angle sum).
y = 42°, because alternate interior angles in parallel lines are equal.
1 mark for each value and 1 mark for each correct reason.

9 / Irregular heptagon [3]

Sum = (7 − 2) × 180° = 900°. The six given angles sum to 760°.
Seventh angle = 900° − 760° = **140°**.
1 mark for 900°. 1 mark for subtracting the six given angles. 1 mark for 140°.

10 / Regular polygon [3]

Exterior angle = 180° − 162° = **18°**.
n = 360° ÷ 18° = **20 sides**.
1 mark for 18°. 1 mark for dividing 360° by 18°. 1 mark for 20.

11 / Ratio of angles [3]

There are 2 + 3 + 4 + 6 = 15 parts. One part is 360° ÷ 15 = 24°.
Angles = **48°, 72°, 96° and 144°**.
1 mark for 15 parts. 1 mark for 24° per part. 1 mark for all four angles.

12 / Polygons meeting at a point [4]

Pentagon angle = (5 − 2) × 180° ÷ 5 = **108°**.
Hexagon angle = (6 − 2) × 180° ÷ 6 = **120°**.
Remaining angle = 360° − 108° − 120° = **132°**.
1 mark for each polygon angle. 1 mark for using 360°. 1 mark for 132°.